Let Σ be a Coxeter diagram, and let T be a subdiagram of Σ such that $\det(\Sigma \setminus T) \neq 0$. A *local determinant* of Σ on a subdiagram T is

$$\det(\Sigma, T) = \frac{\det \Sigma}{\det(\Sigma \setminus T)}.$$

Proposition 1.
If a Coxeter diagram Σ is spanned by subdiagrams $\Sigma_1, \Sigma_2, \dots, \Sigma_l$ having a unique vertex v in common, and no vertex of $\Sigma_i \setminus v$ is adjacent to $\Sigma_j \setminus v$ for $i \neq j$, then

$$\det(\Sigma, v) - 1 = \sum_{i=1}^{l} (\det(\Sigma_i, v) - 1).$$

Proposition 2.
If a Coxeter diagram Σ is spanned by disjoint subdiagrams Σ_1, Σ_2 connected by a unique edge $v_1 v_2$, then

$$\det(\Sigma, \langle v_1, v_2 \rangle) = \det(\Sigma_1, v_1) \det(\Sigma_2, v_2) - a^2,$$

where a is the weight of the edge $v_1 v_2$.

Proposition 3.
If a vertex v of Σ is adjacent to a unique vertex u, then

$$\det(\Sigma, v) = 1 - \frac{a^2}{\det(\Sigma \setminus v, u)},$$

where a is the weight of the edge uv.

[Vin2] E. B. Vinberg, *The absence of crystallographic groups of reflections in Lobachevsky spaces of large dimensions,* Trans. Moscow Math. Soc. 47 (1985), 75–112.